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Combined effect of several interfering signals

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(RA-58)

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SUMMARY

When a site is considered for a new radio or television transmitter the best available assessment of interference expected from existing transmitters operating on or near the same frequency is required. This involves calculation of the combined effect of two or more interfering signals simultaneously present. The information available about each interfering signal is its distribution in time, that is, the percentages of time for which the signal can be expected to exceed certain levels. The resultant of interfering signals is best expressed in the same way, i.e., as the distribution of an equivalent single interfering signal. A general method of finding the resultant of two signal distributions, suitably quantized, is discussed and possible simplifications are considered for distributions approximating to certain standard types.

Correlation has usually been either neglected (so that signals simultaneously present are combined by convolution of their power distributions) or assumed to be complete (so that combination is achieved by power addition of the signals). A tentative method of interpolating between these extremes to allow for actual correlation is discussed in Appendix A; very little relevant experimental information is available.

When an interfering signal has a sufficiently wide range of variation it is usually accurate enough to regard it at any instant as being either interfering or negligible. This is the basis of the 'probability multiplication' procedure hitherto used in Research Department for multiple-interference calculations.

1. INTRODUCTION

The strength of the signal from any particular potentially interfering transmitter may vary rapidly and between wide limits, so that any assessment of the likelihood of interference can only be done statistically. The observable information can be reduced to the form that the signal in question can be expected to be within certain specified limits (or above a certain level) for a certain percentage of the time. We are here concerned with determining the effective resultant of two or more such signals present simultaneously. The data obtained for the variation of the resultant of two interfering signals must be obtained in the same form as the data for the variation of the component signals, so that the process can be repeated as often as is necessary.

The process at present used in Research Department for estimating multiple interference probabilities is theoretically correct if it is assumed that signals vary so widely that, at a given moment, any one signal can be regarded as either interfering even if no other signal were present, or negligible. Many new low-power u.h.f. stations will have to be installed shortly, however, and interference from a nearby station, which only varies in strength by a few decibels, must also be taken into account in future. Any useful method of finding the resultant of interfering signals must take both possibilities into account.

We shall here assume that data are available of the probabilities that any particular unwanted signal shall be above a specific level I_0, 20 or more dB below I_0, or in the intervals 0-1, 1-2, 2-3, 3-4.5, 4.5-6, 6-8, 8-10, 10-12, 12-14, 14-16, 16-18 or 18-20 dB below I_0. We are not concerned with the manner in which such data are obtained. Direct observations would be possible, but would require a great deal of effort. A more practicable alternative would be to observe directly the probabilities that the signal lie within a much smaller number of wide intervals, and interpolate between observed probabilities by assuming that the signal then obeyed some standard law, for instance, that the signal power or its logarithm was normally* distributed. Special, simplified techniques are available, and are discussed, if the signal itself, or its logarithm, is approximately normally distributed over a wide range of levels.

The question of possible correlation of unwanted signals is largely ignored in what follows. The combined effect of two interfering signals distributed normally or 'log-normally' can be calculated for any assumed correlation coefficient, but the correlation coefficient relevant when this resultant is compared with a third such interfering signal is very difficult to assess, unless all the three signals are regarded as completely uncorrelated or completely correlated.

* Throughout this report 'normally distributed' or 'normal distribution' refer to a Gaussian distribution.

In the latter case they are subject to the law of power addition. Here therefore it is assumed that any pairs or sets of signals likely to be highly correlated (because they are associated with stations along paths subject to the same weather conditions, for example) can be combined by power addition, and that the resultant thus obtained is uncorrelated with any other interfering signal or with the resultant of any other group of highly-correlated signals. The effect of correlation is discussed in more detail in Appendix A, particularly for signals which do not vary greatly and can be adequately represented by the equivalent best-fitting normal distribution, or by the 'conservative normal equivalent' distribution discussed below.

In the absence of adequate information on the actual degree of correlation between interfering signals simultaneously present, all that can really be said is that the actual distribution of the resultant of several interfering signals simultaneously present is between the distribution calculated (by convolution) on the assumption that no two of the constituent signals are correlated, and that calculated (by power addition) on the assumption that every pair of constituent signals is completely correlated. If one signal of significant magnitude varies much more widely than the remainder, the resultants obtained on these two extreme assumptions will not differ greatly, and the actual degree of correlation will not be very important. The difference between these two resultants is greatest when the constituent signals have identical distributions.

When two or more interfering signals are present simultaneously, their relative phases are equally likely to have any particular value at any particular instant, and therefore the power of the resultant can be taken as the sum of the powers of the constituent signals. It will therefore be assumed here that any signal distribution is expressed in terms of power before the calculation begins. If the distribution of field strength is 'log-normal' (that is, the ratio of the field strength at any instant to an arbitrary field strength such as 1 μV/m is expressed in decibels by a number which is normally distributed), then the corresponding distribution, expressed in terms of power, is also 'log-normal'.

Now the distribution of the sum of two signals each having a known distribution of arbitrary form can always be obtained approximately by the process of 'quantized convolution' described in Section 2; this process can be repeated indefinitely and is suitably adapted to allow for the fact that the constituent signals are never negative. The sum of two signals can be obtained more simply if both of them can be satisfactorily replaced by the best-fitting equivalent normal distribution. For signals which do not vary greatly, this normal distribution is a good fit, but unfortunately it underestimates the probability that a 'log-normal' distribution reaches high levels. In Section 3 a 'conservative normal equivalent' distribution is derived; reasons are explained for regarding this as a satisfactory approximate substitute for a 'log-normal' distribution having a standard deviation less than about 3 dB. The sum of any number of uncorrelated variables with normal distributions has a normal distribution, with a mean value equal to the sum of the individual means and a standard deviation equal to the square root of the sum of the squares of the individual standard deviations.

The process of quantized convolution discussed in Section 2 appears to be the only method available for assessing the resultant of pairs of signals both members of which cannot be satisfactorily approximated in one of the ways just mentioned. For very widely varying signals, however, such as 'log-normal' distributions having standard deviations greater than about 6 dB, each signal can, as an approximation, be regarded at any instant as either interfering in its own right (i.e. in the absence of any other signals) or negligible. This is the basis of the existing 'probability multiplication' procedure used in reference 1.

2. QUANTIZED CONVOLUTION METHOD FOR FINDING THE DISTRIBUTION OF THE SUM OF TWO SIGNALS WITH ARBITRARY DISTRIBUTIONS

Suppose that, for each of two signals, we are given the probabilities a_λ and b_λ that the signal (necessarily positive) lies within the limits $\lambda I_0/10$ and $(\lambda+1)I_0/10$ for $\lambda = 0, 1 \ldots 9$, and also the probability that it exceeds I_0. We wish to determine the approximate probability distribution of the resultant signal S_1+S_2, on the understanding that if a signal value exceeds I_0, this is all that matters — whether it is $1{\cdot}01 I_0$ or $10^7 I_0$ is irrelevant. We assume that the quantization is sufficiently fine, so that any signal value between $\lambda I_0/10$ and $(\lambda+1)I_0/10$ can be treated as $(\lambda+½)I_0/10$. Then the signal S_1 can be represented geometrically by the histogram of Fig. 1 or algebraically by the expression

$$D(S_1) = X^{½}(a_0 + a_1X + a_2X^2 + a_3X^3 + \ldots + a_9X^9 + A_{10}X^{10}) \tag{1}$$

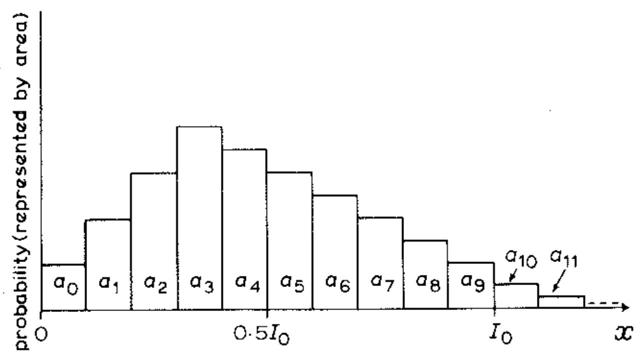

Fig. 1 - Geometrical representation of distribution of signal S_1

where X^n represents a displacement $nI_0/10$ to the right along the X-axis.* The sum of all the probabilities a_0, a_1, etc. is unity, since S_1 certainly has some positive value; the symbol A_{10} in equation (1) denotes $a_{10}+a_{11}+a_{12}+...$ since any value of S_1 above I_0 is for our present purpose equivalent to any other. This well-known 'generating function' representation of a quantized probability distribution has the important advantage that if the second signal S_2 is represented by

$$D(S_2) = X^{1/2}(b_0+b_1X+b_2X^2+ ... +b_9X^9+ B_{10}X^{10}) \quad (2)$$

(where B_{10} is the sum of all coefficients b_{10}, b_{11}, b_{12} etc. which are associated with the second signal S_2), then the corresponding probability distribution for the sum $S_1 + S_2$ (neglecting correlation) is represented by the algebraic product of the right hand sides of equations (1) and (2), namely

$$D(S_1 + S_2) = C_1X + C_2X^2 + C_3X^3 + ... + C_9X^9 + C_{10}X^{10} + D_{11}X^{11} \quad (3)$$

where

$$C_i = \sum_{r=0}^{i-1} a_r b_{i-1-r} \quad (i = 1, 2 ... 21) \quad (4)$$

$$D_{11} = C_{11} + C_{12} + ... + C_{21}$$

From equation (3) we conclude that the probability that (S_1+S_2) lies between $(i-\frac{1}{2})I_0/10$ and $(i+\frac{1}{2})I_0/10$ is C_i for $i = 1, 2, 3 ... 10$, and that the probability of $(S_1 + S_2)$ exceeding I_0 is $D_{11} + \frac{1}{2}C_{10}$; we assume that signal levels associated with the term $C_{10}X^{10}$ in equation (3) are equally likely to be above or below I_0.

Now equation (3) has a form such that we could repeat this process to find the resultant distribution of $(S_1 + S_2)$ and a third signal S_3 expressed in a form analogous to (1), and so on. By using a sufficiently fine quantization interval, we could determine (S_1+S_2) as accurately as we wished, but refining the quantization increases the computational labour (or machine time) required to calculate the distribution of S_1+S_2.

The basic reason why this process works is that

$$X^mX^n = X^{m+n} \quad (5)$$

and that the number of different relevant values of $(m+n)$ which occur in the algebraic product [equation (3)] is no greater than for each factor [equations (1) and (2)].

For actual signal distributions, the linear quantization assumed above is too coarse (for a given number of quantization intervals) both near the interference level and at low signal levels, but if we have an arbitrary set of quantized levels in an attempt to remedy this, the number of different possible values of $(m+n)$ may increase violently, so that the process of multiplication cannot be satisfactorily repeated. Here we shall assume that there are 14 quantized levels, as specified in Table 1 below.

TABLE 1

Quantization Ranges

Range (dB below I_0)	Quantized Value Representing Range	Probability Symbol
At or above I_0	$>I_0$	A_{13}
0-1	$0.89I_0$	A_{12}
1-2	$0.71I_0$	A_{11}
2-3	$0.56I_0$	A_{10}
3-4.5	$0.42I_0$	A_9
4.5-6	$0.30I_0$	A_8
6-8	$0.20I_0$	A_7
8-10	$0.13I_0$	A_6
10-12	$0.08I_0$	A_5
12-14	$0.05I_0$	A_4
14-16	$0.03I_0$	A_3
16-18	$0.02I_0$	A_2
18-20	$0.01I_0$	A_1
More than 20	0	A_0

In a notation analogous to that used for equation (1), the signal S_1 can therefore be represented by the expression

$$D(S_1) = A_{13}X^{100} + A_{12}X^{89} + A_{11}X^{71} + A_{10}X^{56} + A_9X^{42} + A_8X^{30} + A_7X^{20} + A_6X^{13} + A_5X^8 + A_4X^5 + A_3X^3 + A_2X^2 + A_1X + A_0 \quad (6)$$

where X^n denotes a displacement of $0.01\ I_0n$. If a second signal S_2 is represented by a similar expression, with the same powers of X but different coefficients, say B_i, then (S_1+S_2) is strictly speaking correspondingly represented by the algebraic product of (6) with the similar expression for S_2.

Now this algebraic product involves all the powers of X which can be obtained by adding two of the powers listed in equation (6), such as 56 + 30 = 86,

* The use of an exponent-type suffix to designate displacement is justified later.

20 + 13 = 33, 71 + 8 = 79, and so on. The approximation we shall make is to replace any of the numbers 86, 33, 79 etc. by the nearest index occurring in equation (6); when we are in doubt, the higher of the two possible values for this index will be chosen. With this approximation, we can represent $S_1 + S_2$ in the same form, namely

$$D(S_1 + S_2) = C_{13}X^{100} + C_{12}X^{89} + C_{11}X^{71} + C_{10}X^{56} + C_9X^{42} + C_8X^{30} + C_7X^{20} + C_6X^{13} + C_5X^8 + C_4X^5 + C_3X^3 + C_2X^2 + C_1X + C_0 \quad (7)$$

where the formulae for the coefficients C_i of the resultant $S_1 + S_2$ in terms of the corresponding coefficients associated with S_1 and S_2 themselves are given and briefly explained in Appendix B.

The process of quantized convolution described above can be applied to a pair of signals having arbitrary distributions: the only limitation is due to the quantization. This quantization can be refined at the expense of greater labour of calculation (or greater machine time) which may not be justified since field strengths are notoriously difficult to measure to a high degree of accuracy. But there are certain distributions for which the convolution process is greatly simplified. These cases are considered in Section 3.

3. DISTRIBUTIONS FOR WHICH THE SUMMATION PROCESS CAN BE SIMPLIFIED

For statistical investigations in general, the first simplifying approximation worth considering is usually to replace the given distribution by the best-fitting equivalent normal distribution. There are well-known techniques for doing this. There is often good theoretical justification for expecting that this equivalent normal distribution will represent well the given distribution.

Hitherto it has usually been assumed that field strength in general tends to be 'log-normally' distributed, with some reservations that the probability of very large signals may be significantly overestimated by this assumption. If the given distribution can be regarded as varying violently, in a manner comparable with a 'log-normal' distribution having a standard deviation of 6 dB or more, the obvious simplifying approximation is to regard the signal in question as either negligible, or interfering even if all other signals simultaneously present are ignored. If the given distribution can be regarded as comparable to a 'log-normal' distribution having standard deviation between say 3 and 6 dB, there appears to be no short-cut to the process of quantized convolution discussed in Section 2 when the signal having this distribution is present at the same time as other signals. If the given distribution is comparable to a 'log-normal' distribution having standard deviation less than about 3 dB, the given distribution could be replaced by its best-fitting normal equivalent, but this procedure has the disadvantage that the probability of high signal strengths near the level of interference which we wish to avoid tends to be underestimated. A preferable alternative therefore appears to be to replace the given distribution by the 'conservative normal equivalent' distribution described next.

The 'conservative normal equivalent' of a 'log-normal' distribution of sufficiently low standard deviation is defined to be the normal distribution which is such that the interference level I_0 and an arbitrary lower level I_1 occur with the same probability in both distributions. Since an actual signal is essentially positive, the 'conservative normal equivalent' concept is not useful unless the occurrence of a negative signal in this distribution is extremely improbable. If in both distributions I_0 is m standard deviations above the mean and I_1 is n standard deviations below it,

$$(m + n)\sigma = 10 \log_{10}(I_0/I_1)$$
$$(m + n)\sigma_c = I_0 - I_1 \quad (8)$$

where σ is the 'log-normal' standard deviation, σ_c is the 'conservative normal equivalent' standard deviation, and the coefficient of $\log_{10}(I_0/I_1)$ is 10 and not 20 since the signals are assumed expressed in terms of power and not field strength as already mentioned. We shall assume that zero must be $3\sigma_c$ below the mean of the 'conservative normal equivalent' distribution for negative values to be sufficiently improbable, and we shall consider in detail only the case when $m = n = 2$. The maximum permissible value of σ_c is then $0{\cdot}2I_0$ and $I_1 = 0{\cdot}2I_0$, while $\sigma = 1{\cdot}747$ dB. The 'conservative normal equivalent' estimate of the probability of any signal level between I_0 and I_1 is greater than the estimate associated with the 'log-normal' distribution. The mean of the 'conservative normal equivalent' distribution is $0{\cdot}6I_0$ in this case, 2·22 dB below I_0 and 1·28 dB above the mean of the 'log-normal' distribution. If the log-normal standard deviation σ is less than 1·747 dB, the 'conservative normal equivalent' standard deviation σ_c is given by

$$4\,\sigma_c/I_0 = 1 - 10^{-0\cdot4\sigma} \quad (9)$$

If $\sigma > 1{\cdot}747$ dB, σ_c must remain at its maximum permissible value of $0{\cdot}2I_0$ or negative 'conservative normal equivalent' values become excessively probable, and the discrepancy between the two distributions rapidly increases. We have therefore somewhat arbitrarily concluded that 3 dB is about the highest value of σ for which the idea of a 'conservative normal equivalent' distribution is useful. When this idea is useful, however, any number of signals replaceable by 'conservative normal equivalent' distributions can be combined by adding the means of these distributions and taking the resultant standard deviation as the square root of the sum of the squares of the individual 'conservative normal equivalent' standard deviations.

4. QUANTIZATION IN TERMS OF FIXED PROBABILITY INTERVALS

It is worth considering whether quantization should be carried out in terms of fixing the probability interval instead of fixing the range of signal strength to be regarded as a quantum. Thus instead of estimating the probability that a signal be 0-1 dB, 1-2 dB, 2-3 dB, 3-4·5 dB, etc. below I_0, we say that the signal is between I_0 and α_1 dB below I_0 for 1% of the time (and taken as $\frac{1}{2}\alpha_1$ dB below I_0 for that time), between α_1 dB below I_0 and α_2 dB below I_0 for 2% of the time [and taken as $\frac{1}{2}(\alpha_1 + \alpha_2)$ dB below I_0 for that time], and so on. This possibility has not been discussed here, because it means that when the determination of the resultant of two signals is carried out as indicated in Section 2 and Appendix B, the powers of X involved are different for each pair of signals to be combined, whereas the coefficients A_1, A_2 etc. and B_1, B_2 etc. are fixed. This has the disadvantage that Table 1 and equations (B2) to (B15) would have to be formulated afresh (at least within the computer) for each pair of signals summed, and it is not clear what compensating advantage can be derived from the constancy of the coefficients A_1, B_2 etc.

5. CONCLUSIONS

A procedure for finding the resultant of two arbitrary interfering signals has been devised which is repeatable and takes account of the fact that the constituent signals are essentially positive and not necessarily suitable for quantization in equal linear steps. The quantization intervals can easily be adjusted if necessary.

This procedure can be greatly simplified if some signals can be satisfactorily replaced by suitable 'conservative normal equivalents' which always overestimate the probability of occurrence of signals near the interference level. Widely-varying signals can usually be treated as either interfering in the absence of all other signals, or negligible.

The convolution procedure discussed is strictly applicable only to uncorrelated signals. Completely correlated signals should instead be combined by the law of power addition. Insufficient experimental information is at present available to decide how in practice to interpolate satisfactorily between these two extremes, but it is tentatively suggested that interfering signals should be divided into highly-correlated groups combined by power addition, and that otherwise correlation should be ignored.

6. REFERENCE

1. Calculation of the field strength required for a television service, in the presence of co-channel interfering signals: Effect of multiple interfering sources. BBC Research Department Report No. RA-12/2, Serial No. 1968/43.

APPENDIX A

The Effect of Correlation

In the main text, we have assumed that two distributions simultaneously present should normally be combined by the 'quantized convolution' process discussed in Section 2. Such convolution in fact neglects any correlation between the two distributions. At the other extreme, we can assume complete correlation between the two distributions, and combine them by the law of power addition.

Determination of the correlation coefficient between any pair of unwanted signals is not easy. Cases do occur of high correlation coefficients (of the order of 0·9) between the hourly means of signals from stations which are either near the receiver or have similar paths to the receiver. But the short-term correlation coefficient (which in effect measures the relative behaviour of instantaneous deviations from these hourly means) is usually small. Strictly speaking, it is simultaneous powers of unwanted signals which must be added to obtain the corresponding instantaneous level of multiple interference, and it is the probability of a particular instantaneous value of each unwanted signal which has to be estimated from the statistics of its distribution.

For signals which are (or are nearly) normally distributed, the effect of correlation is easily appreciated in general terms. If x_1 is normally distributed about mean M_1 with standard deviation σ_1 and x_2 is normally distributed about mean M_2 with standard deviation σ_2, and the correlation coefficient between the signals is ρ_{12}, then $(x_1 + x_2)$ is normally distributed about mean $(M_1 + M_2)$ with standard deviation Σ_{12} given by

$$\Sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2 \qquad \text{(A1)}$$

Hence the general effect of a positive correlation is to increase the standard deviation of the sum $(x_1 + x_2)$ to Σ_{12} instead of $(\sigma_1^2 + \sigma_2^2)^{1/2}$, the value obtained in the absence of correlation. If $\rho_{12} = +1$, Σ_{12} becomes $(\sigma_1 + \sigma_2)$. If $\sigma_1 = \sigma_2$ (the worst case) $\Sigma_{12}(\rho_{12} = 1)$ is 41% above its value for $\rho_{12} = 0$. Hence by neglecting (a positive) correlation in this case we shall make no error in determining the mean of the sum, but we shall underestimate the standard deviation by as much as 41% if σ_1 and σ_2 are nearly equal, and we shall correspondingly underestimate the probability that the sum shall exceed a given level higher than the mean. Again, if we assume that x_1 and x_2 are completely correlated, and determine the distribution of the sum by power addition, we shall overestimate Σ_{12} relative to its correct value given by (A1), and we shall correspondingly overestimate the probability that the sum will exceed a given level higher than the mean.

More generally, if we have several quantities of which ξ_i and ξ_j are a typical pair, normally distributed about mean zero with standard deviations σ_i, σ_j and such that the correlation coefficient between the two distributions is ρ_{ij}, then the distribution of $(\xi_1 + \xi_2 + \ldots + \xi_n)$ is normal with mean zero and standard deviation Σ where

$$\Sigma^2 = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots + \sigma_n^2) + 2\rho_{12}\sigma_1\sigma_2 + 2\rho_{13}\sigma_1\sigma_3 + \ldots + 2\rho_{n-1,n}\sigma_{n-1}\sigma_n \qquad \text{(A6)}$$

If all the ρ_{ij} are zero,

$$\Sigma^2 = \Sigma_0^2 = (\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots + \sigma_n^2)$$

If all the ρ_{ij} are +1, $\Sigma = \Sigma_1 = \sigma_1 + \sigma_2 + \ldots + \sigma_n$

Here again, the effect of correlation appears to be simply to alter the resultant standard deviation Σ. If Σ_0 is the value of Σ obtained when correlation is neglected, and the corresponding value of Σ obtained when full account is taken of correlation is $(1+\lambda_1)\Sigma_0$, then the probability calculated as that for the sum to be between α and β on the assumption of zero correlation is really the probability for the sum to be between $(1 + \lambda_1)\alpha$ and $(1 + \lambda_1)\beta$. The difficulty is to estimate λ_1 correctly. All we really know is that if non-negative correlation exists between each pair of unwanted signals, λ_1 is between the value zero appropriate to uncorrelated signals, and the maximum value appropriate when each pair of signals is completely correlated; this maximum value is λ_{max} where

$$1+\lambda_{max} = (\sigma_1 + \sigma_2 + \sigma_3 + \ldots + \sigma_n)/(\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2)^{1/2} \qquad \text{(A2)}$$

This suggests that two resultant distributions D_0 and D_1 should always be worked out on the respective assumptions of zero and unity correlation between any pair of constituent signals. The actual distribution is then bracketed between D_0 and D_1. Any normal or nearly normal distribution can be combined for D_0 by adding means and taking the resultant standard deviation as the square root of the sum of the constituent standard deviations; this resultant is then combined by quantized convolution with the remaining non-normal constituent signals. Distribution D_1 is calculated as if there were total correlation between each pair of constituent signals. Any normal or nearly normal distributions can be combined by adding means and standard deviations, and the resultant of these is then combined by power addition with the remaining non-normal constituent signals.

One method of reducing the uncertainty because of partial correlation might be to group together signals which are likely to be highly correlated (for example, because the signal paths are subject to similar weather conditions) and to treat signals within any such group, say G_r, as completely correlated. Then the sum of the signals within group G_r can be regarded as a distribution of type D_1, and the resultant R_r obtained in an analogous manner. The sum of the various resultants R_r and the interfering signals which did not belong to any of the groups G_r, on the other hand, can be regarded as an uncorrelated distribution of type D_0.

APPENDIX B

Formulae for Coefficients in a Quantized Convolution Product

We have to consider here the algebraic product of an expression

$$S_1 = A_{13}X^{100} + A_{12}X^{89} + A_{11}X^{71} + A_{10}X^{56} + A_9X^{42}$$
$$+ A_8X^{30} + A_7X^{20} + A_6X^{13} + A_5X^8 + A_4X^5 + A_3X^3$$
$$+ A_2X^2 + A_1X + A_0 \qquad \text{(B1)}$$

with a similar expression S_2 in which the same powers of X occur but different coefficients B_0, B_1, B_2, ... B_{13} occur. In order that the process of algebraic multiplication may be repeatable, it is necessary to express the sum, $S_1 + S_2$, of the two signals in the same form with different coefficients C_0, C_1, C_2, C_{13}, as in equation (7) of the main text.

Table 1 shows how the powers of X arising when the algebraic product is formed can be rounded off and replaced by the nearest power occurring in equation (B1). This nearest value is estimated conservatively when there is any doubt. The rows and columns of Table 1 are numbered according to the suffices of the coefficients of equation (B1). The second row specifies the appropriate quantized values of signal S_2 while the second column specifies the appropriate quantized values of signal S_1. The tabular entry is the corresponding quantized value appropriate to the signal $(S_1 + S_2)$. The formulae for the coefficients C_0, C_1, C_2 ... C_{13} associated with $(S_1 + S_2)$ in equation (7) of the main text are given below. They are mainly derived by repeated use of Table 1. The only point of difficulty is the initial term $(A_{13} + B_{13} - A_{13}B_{13})$ in the expression for C_{13}.

TABLE 1

Quantized values for Powers of X in Signal Products

Signal S_2	Column No. / dB below I_0 / Quantized Value		13	12	11	10	9	8	7	6	5	4	3	2	1	0
	dB below I_0		$>I_0$	0-1	1-2	2-3	3-4.5	4.5-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	over 20
	Quantized Value		I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.20I_0$	$0.13I_0$	$0.08I_0$	$0.05I_0$	$0.03I_0$	$0.02I_0$	$0.01I_0$	0
Row Number	Signal S_1 dB below I_0	Signal S_1 Quantized Value	The tabular entry is the quantized value of $(S_1 + S_2)$ when S_1 has the quantized value at the left of the same row, and S_2 the quantized value at the head of the same column.													
13	$>I_0$	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0
12	0-1	$0.89I_0$	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	I_0	$0.89I_0$	$0.89I_0$	$0.89I_0$	$0.89I_0$	$0.89I_0$
11	1-2	$0.71I_0$	I_0	I_0	I_0	I_0	I_0	I_0	$0.89I_0$	$0.89I_0$	$0.89I_0$	$0.71I_0$	$0.71I_0$	$0.71I_0$	$0.71I_0$	$0.71I_0$
10	2-3	$0.56I_0$	I_0	I_0	I_0	I_0	I_0	$0.89I_0$	$0.71I_0$	$0.71I_0$	$0.71I_0$	$0.56I_0$	$0.56I_0$	$0.56I_0$	$0.56I_0$	$0.56I_0$
9	3-4.5	$0.42I_0$	I_0	I_0	I_0	I_0	$0.89I_0$	$0.71I_0$	$0.71I_0$	$0.56I_0$	$0.56I_0$	$0.42I_0$	$0.42I_0$	$0.42I_0$	$0.42I_0$	$0.42I_0$
8	4.5-6	$0.30I_0$	I_0	I_0	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.56I_0$	$0.42I_0$	$0.42I_0$	$0.30I_0$	$0.30I_0$	$0.30I_0$	$0.30I_0$	$0.30I_0$
7	6-8	$0.20I_0$	I_0	I_0	$0.89I_0$	$0.71I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.30I_0$	$0.30I_0$	$0.20I_0$	$0.20I_0$	$0.20I_0$	$0.20I_0$
6	8-10	$0.13I_0$	I_0	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.30I_0$	$0.20I_0$	$0.20I_0$	$0.20I_0$	$0.13I_0$	$0.13I_0$	$0.13I_0$
5	10-12	$0.08I_0$	I_0	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.20I_0$	$0.20I_0$	$0.13I_0$	$0.13I_0$	$0.13I_0$	$0.08I_0$	$0.08I_0$
4	12-14	$0.05I_0$	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.30I_0$	$0.20I_0$	$0.13I_0$	$0.13I_0$	$0.08I_0$	$0.08I_0$	$0.05I_0$	$0.05I_0$
3	14-16	$0.03I_0$	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.20I_0$	$0.20I_0$	$0.13I_0$	$0.08I_0$	$0.05I_0$	$0.05I_0$	$0.05I_0$	$0.03I_0$
2	16-18	$0.02I_0$	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.20I_0$	$0.13I_0$	$0.13I_0$	$0.08I_0$	$0.05I_0$	$0.05I_0$	$0.03I_0$	$0.02I_0$
1	18-20	$0.01I_0$	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.20I_0$	$0.13I_0$	$0.08I_0$	$0.05I_0$	$0.05I_0$	$0.03I_0$	$0.02I_0$	$0.01I_0$
0	over 20	0	I_0	$0.89I_0$	$0.71I_0$	$0.56I_0$	$0.42I_0$	$0.30I_0$	$0.20I_0$	$0.13I_0$	$0.08I_0$	$0.05I_0$	$0.03I_0$	$0.02I_0$	$0.01I_0$	0

If the first signal S_1 is above I_0, then $S_1 + S_2$ will necessarily be above I_0, so that the probability that $S_1 + S_2$ is above I_0 is simply A_{13}. Similarly, the probability that $S_1 + S_2$ is above I_0 because S_2 is above I_0 is simply B_{13}. But this has counted twice the case when S_1 and S_2 are both above I_0, for which the probability is $A_{13}B_{13}$.

The full results are:

$$C_{13} = (A_{13} + B_{13} - A_{13}B_{13}) + (A_5 + A_6 + \dots + A_{12})B_{12} + (A_8 + A_9 + A_{10} + A_{11} + A_{12})B_{11} + (A_9 + A_{10} + A_{11} + A_{12})B_{10} + (A_{10} + A_{11} + A_{12})B_9 + (A_{11} + A_{12})B_8 + A_{12}(B_5 + B_6 + B_7) \quad \text{(B2)}$$

$$C_{12} = (A_0 + A_1 + \dots + A_4)B_{12} + (A_5 + A_6 + A_7)B_{11} + A_8B_{10} + A_9B_9 + A_{10}B_8 + A_{11}(B_5 + B_6 + B_7) + A_{12}(B_0 + B_1 + \dots + B_4) \quad \text{(B3)}$$

$$C_{11} = (A_0 + A_1 + \dots + A_4)B_{11} + (A_5 + A_6 + A_7)B_{10} + (A_7 + A_8)B_9 + A_9(B_7 + B_8) + A_{10}(B_5 + B_6 + B_7) + A_{11}(B_0 + B_1 + \dots + B_4) \quad \text{(B4)}$$

$$C_{10} = (A_0 + A_1 + \dots + A_4)B_{10} + (A_5 + A_6)B_9 + (A_7 + A_8)B_8 + A_8B_7 + A_9(B_5 + B_6) + A_{10}(B_0 + B_1 + \dots + B_4) \quad \text{(B5)}$$

$$C_9 = (A_0 + A_1 + \dots + A_4)B_9 + (A_5 + A_6)B_8 + A_7B_7 + A_8(B_5 + B_6) + A_9(B_0 + B_1 + \dots + B_4) \quad \text{(B6)}$$

$$C_8 = (A_0 + A_1 + \dots + A_4)B_8 + (A_4 + A_5 + A_6)B_7 + A_6B_6 + A_7(B_4 + B_5 + B_6) + A_8(B_0 + B_1 + \dots + B_4) \quad \text{(B7)}$$

$$C_7 = (A_0 + A_1 + A_2 + A_3)B_7 + (A_3 + A_4 + A_5)B_6 + A_6(B_3 + B_4 + B_5) + A_7(B_0 + B_1 + B_2 + B_3) \quad \text{(B8)}$$

$$C_6 = (A_0 + A_1 + A_2)B_6 + (A_2 + A_3 + A_4)B_5 + A_4B_4 + A_5(B_2 + B_3 + B_4) + A_6(B_0 + B_1 + B_2) \quad \text{(B9)}$$

$$C_5 = (A_0 + A_1)B_5 + (A_2 + A_3)B_4 + A_4(B_2 + B_3) + A_5(B_0 + B_1) \quad \text{(B10)}$$

$$C_4 = (A_0 + A_1)B_4 + (A_1 + A_2 + A_3)B_3 + A_3(B_1 + B_2) + A_4(B_0 + B_1) \quad \text{(B11)}$$

$$C_3 = A_0B_3 + A_1B_2 + A_2B_1 + A_3B_0 \quad \text{(B12)}$$

$$C_2 = A_0B_2 + A_1B_1 + A_2B_0 \quad \text{(B13)}$$

$$C_1 = A_0B_1 + A_1B_0 \quad \text{(B14)}$$

$$C_0 = A_0B_0 \quad \text{(B15)}$$

Now although the appearance of some of equations (B2) to (B15) is somewhat formidable, it is in principle very little more difficult to write a computer programme to give the C's when the A's and B's are given (or have previously been found) than to write a programme for the ordinary algebraic-multiplication type of convolution considered in relation to equation (3) of the main text. Equations (B2) to (B15) as they stand appear to be a satisfactory compromise between a quantization which is too coarse and one which is so fine that excessive machine time is required for evaluating the C's when the A's and B's are known. But it would not be at all difficult to adjust Table 1 and equations analogous to (B2), (B3) etc. to cope with any different quantization which might appear to be preferable.

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